Graph Models of Social Media Network As Applied to Facebook and Facebook Messenger Groups

Tsok Samuel Hwere*, Hosea Yakubu and Rwat Solomon Isa***
*Department Of Mathematics Faculty of Natural and Applied Sciences Plateau State University Bokkos, Plateau State, Nigeria.
** School of Applied Sciences, Federal Polytechnic Of Oil, Bonny Island, River State Nigeria
tsoksam@gmail.com

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Abstracts
Graphs are extensively used to model social structures based on different kinds of relationships between people or groups of people. Facebook can conveniently be modelled using graphs, the connection between individuals in Facebook media can be described using graphs, ranging from two individuals connected, communication between different Facebook friends and Facebook messenger groups, this paper comfortably explained this idea with graphs models. The idea of influences or directed graphs, acquaintance or undirected graphs, collaboration graphs and adjacent matrix as it's applied to Facebook groups has been presented.

Keywords: *Graph Models, Social Media Network, Facebook groups, collaboration graphs, influences graph, Acquaintances graph.*

1.0 INTRODUCTION

Graphs are discrete structures consisting of vertices and edges that connect these vertices. There are different kinds of graphs, depending on whether edges have directions, whether multiple edges have directions and can connect the same pair of vertices, or whether loops are allowed. Problems in almost every conceivable discipline can be solved using graph models.

Graphs are used in a wide variety of models and are extensively used to model social structures based on different kinds of relationships between people or groups of people [5]. This kind of graph models, individuals or organizations is represented by vertices, relationships between individuals or organizations are represented by edges.

The study of social networks is an extremely active multidisciplinary area and many different types of relationships between people have been studied using them. With these, this paper decided to work on graph models of social media networks relating it to different Facebook groups.

Social media is frolicking an imperative role in circadian practices of communication with people all over the world. It has been extensively emerged among popular with young adults over the last decade. Growth in web-based technologies has widened the use of interactive social media that enables users to upload images and videos on the Internet. Social media networks are very much auxiliary to connect with our friends. Nowadays, whenever friends meet, they first confirm whether they use social media networks like Facebook, WhatsApp,

tweeters, Zooms, Google meets and etc. Many find it as God's gift since they are to connect even to their childhood friends.
In previous years, there was not much communication facility. People were not able to get connected to their friends and relatives often. They felt it was a big process even communicates for a little time. The cost of communication also was very expensive.

1.1 Graph Theory

Graphs are discrete structures consisting of vertices and edges that connect these vertices. There are different kinds of graphs, depending on whether edges have directions or not, whether multiple edges can connect the same pair of vertices, and whether loops are allowed. Problems in almost every conceivable discipline can be solved using graph models [5]. This paper illustrates how graphs models in a variety of areas particularly in social media network as related to Facebook and Facebook messenger. As we all know, showing any data in pictures gives a clear understanding of the usage of words. In general, pictorial
Representation has been referred to as graphs. Graph Theory plays a major role in every field. In Computer Science Engineering, it has a vast application. It plays a major role in both software and hardware usage. For Example, in software, it is used in the data flow diagram, graphical design, network designing. In hardware, it is used in the data structure, image processing, web designing, etc

1.2 Social Media Networks

Nowadays, Technology is moving on its rising slope. We find many communication networks. Within seconds, we can share our status, photos, and messages. Besides, we are also able to have group chats.
In no time, we can communicate with many recipients at a time, with the help of groups. Not only for friendly chats. Many companies form groups with their employees and using that they can share their communication content with ease. Education also gets improved with the help of these communication networks.
There are many social media networks, using them; one is able to communicate by text, voice, pictures, and videos and so on. Many discover it comfortable to use Facebook. Different groups like NGO's, Associations, and Business partners, families, teammates and as well as in academic fields. Tertiary institutions have since started enjoying the full benefits of Facebook and Facebook messengers through different platforms such as level platform, course mate platform, Departmental platform e.t.c. The platforms enable students to share information regarding fixed lectures, test date, examination time tables, tutorials and many more benefits numerous to mention, these enhance easy access to each other's resources.
In this paper, we would like to model Facebook and Facebook messengers' groups using related ideas in graph theory.

1.3 Why do we have different groups of friends on Facebook?

There is a natural tendency of individuals to form friendships with others who are similar to them, this is an ancient proverb, which said that "birds of feather flock together" with roots traced back to 1545 when William Turner, emphases that the similarities could vary from belonging to the same race or geographical location, to sharing similar educational background or behaviour. On the other hand, people also change their mutable characteristics to align themselves more closely with the characteristics of their friends. Examples of such mutable characteristics are behaviour, health, attitude, etc. In addition to these factors, there is a high likelihood of dissolution of friendships between dissimilar individuals. So in a

typical social network, there is a bias in friendships between individuals with similar characteristics. This phenomenon is termed homophily.

1.4 Features of Facebook

Facebook is the most popular social networking website on the internet with more than 1.15 billion active users in March 2013 [3]. Facebook is a User-friendly, it takes less time for communication, and has good privacy service, Online calling facility, can set the lock to the application, easy to use, even a novice mobile user can use it. It instantly sends message to anywhere in the world, it does not have any advertisements on display screen. Its larger group is the public friends feature and other group's features which are as:

- Acquaintances friends
- Friends
- Friends except
- Specific friends
- Close friends
- Only me

2.0 Graph Theory Terminology

2.1 Graph: A graph-generally denoted G (V, E) or G= (V, E) - consists of the set of vertices V unitedly with a set of edges E. The number of vertices in a graph is normally denoted n while the number of edges is normally denoted m.
2.2 Edge: An edge is a line at which vertices are connected in the graph. Edges are denoted by E= (U, V) it is a pair of two vertices.
2.3 Null graph: A graph G= (V (G), E (G)) is a Null Graph if there are no edges in the graph, that is |E (G)|=0.
2.4 Complete graph: A simple graph G= (V, E) with n mutually adjacent vertices is called a complete graph G and it is denoted by K_n or A simple graph G= (V, E) in which every vertex in mutually adjacent to all other vertices is called a complete graph G.

2.5 The Degree of a vertex: Number of edges that are incident to the vertex is called the degree of the vertex.
2.6 Regular graph: In a graph if all vertices have the same degree (incident edges) k than it is called a regular graph.
2.7 Cycle graph: A simple graph G= (V, E) with n vertices (n≥3) n edges is called a cycle graph.
2.8 Source vertex: Is a vertex with in-degree zero. Also, a vertex that has many out-degree with no number of in-degree.
2.9 Directed graph: A directed graph in which represented by an ordered pair of two vertices, e.g. (V denotes an edge from V_i to V_j (from the first vertex to the second vertex).
2.91 Disconnected graph: A graph G is said to be disconnected if there exist two vertices in G such that no edge in G has those vertices as endpoints.

3.0 Graph Models of Facebook and Facebook messengers

Graphs are used in a wide variety of models. We are going to dwell on the description of how to construct Graph models of communications in social media networks. We shall be describing some diverse graph models for some interesting applications in Facebook and

Facebook messenger groups. The reason behind using the graphical model in social networks is to describe the network more elaborately and systematically. This graph model allows us to describe the properties of the network elaborately and distinctly. The relation between the individuals can be mapped using the graphical method and how one can access the resources of other users can be described.

Communication needs both the sender and receiver. Facebook uses two types of communications like single sender many receivers. Many sender single receiver, many senders many receivers and so on. In Facebook, first, it suggests the pictures and names of the persons who are all using Facebook like us, especially people you may know due to the nature of your work, relations, business partners or friends. These can be compared with the set of all vertices.

3.1 Edge

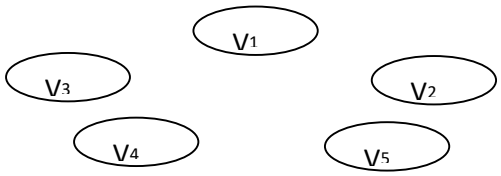
In a Facebook messenger when two people are friends and in a chat, it means they are connected. The relation between them can simply be model by a graph with an edge connecting two vertices V_1 and V_2 as shown in Figure 1. This kind of graph is known as the collaboration graph, where $V_1$and $V_2$are related and are working together in a particular way

Figure 1: A model of a collaboration graph connecting two people as related to a Facebook messenger chat between two persons

3.2 Null Graph

Let us consider 5 persons using Facebook messenger group. Then, the graph consists of 5 vertices as shown in Figure 2. When all the persons are idle, there is no communication and we say is related to a null graph, as shown in Figure 2, a Facebook messenger group that is not active. In graph theory, if no edges are connecting the vertices, we say the group is idle.

Adjacency Matrix

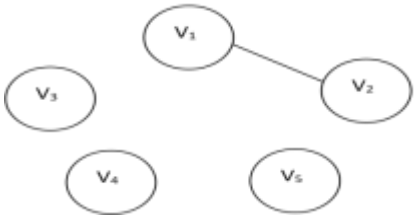


	V_1	V_2	V_3	V_4	V_5
V_1	0	0	0	0	0
V_2	0	0	0	0	0
V3	0	0	0	0	0
V4	0	0	0	0	0
V5	0	0	0	0	0

Figure 2: A model of null graph related to Facebook messenger Inactive group

3.3 Incomplete Graph

In a group chat when only two persons are active in a chat while the others are inactive, it means the two are connected while all the other members of the group are inactive. In graph theory, this is described as an incomplete graph model. For example, we consider an edge between V_1 and V_2, as connected while all other vertices not connected shown in Figure 3.



Adjacency matrix

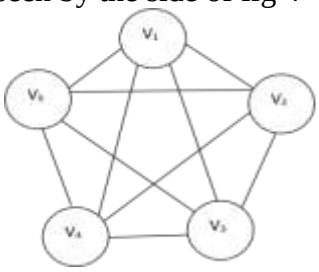
	V1	V2	V3	V4	V5
V1	0	1	0	0	0
V2	1	0	0	0	0
V3	0	0	0	0	0
V4	0	0	0	0	0
V5	0	0	0	0	0

Figure 3: A model of incomplete graph related to a Facebook messenger group of five, when only two people are active.

3.4 Acquaintance Graph (Complete Graph)

This refers to friends who are connected because they know each other and have form a group chat and may have particular related issues, politics religion, academics, etc. When all the persons need to communicate and are communicating at a time, then a Facebook messenger group is formed and used [7].

A Facebook messenger group in which both the admin and all members of the group are allowed to send and receive messages can also be applicable to a complete graph as in Figure 4. Two vertices which are incident with a common edge are said to be adjacent. In a complete graph given in Figure 4, every vertex is adjacent to each other. Facebook messenger chat, where every person is connected to every other person, this can also be known as a complete active group. Again referring to a group chat of 5 persons, where every person is connected to 4 other persons. In Graph Theory, this can be referred to as a regular graph, in which every vertex has the same degree. Figure 4 is an example of such a regular graph and is also called an acquaintanceship graph. The adjacency matrix of the graph is presented as seen by the side of fig 4



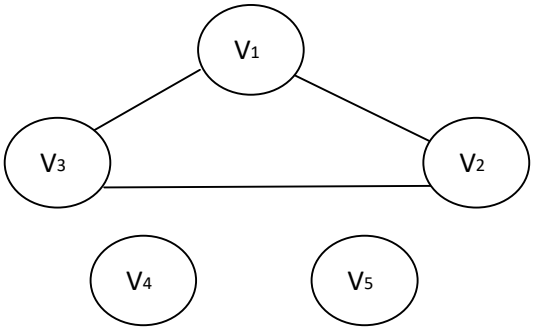
Adjacency matrix

	V1	V2	V3	V4	V5
V1	0	1	1	1	1
V2	1	0	1	1	1
V3	1	1	0	1	1
V4	1	1	1	0	1
V5	1	1	1	1	0

Figure 4: Model of acquaintanceship graph (regular or complete graph) as applied to Facebook messenger group of five active members

3.5 Cycle Graph

Once again, referring to a suppose three-person are active in a group chat, while the others are inactive their communication graph model is related to Cycle graph as in fig 5, V_1, V_2, and V_3 are active while V_4 and V_5 are not active, this means that there is communication between three persons only and the other two persons are idle this can be referred to cycle graph with 3 vertices as shown in Figure 5. The adjacency matrix is presented as in fig 5

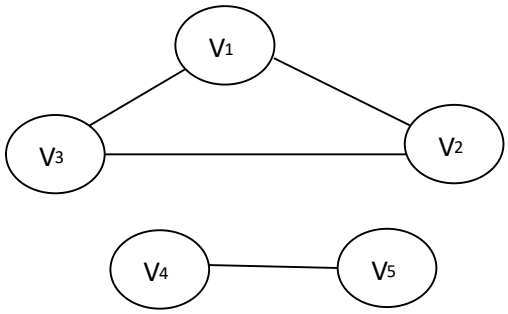


	V_1	V_2	V_3	V_4	V_5
V_1	0	1	1	0	0
V_2	1	0	1	0	0
V_3	1	1	0	0	0
V_4	0	0	0	0	0
V_5	0	0	0	0	0

Figure 5: A model of cycle graph applicable to Facebook messenger group of five but three active members and two idle. **Adjacency matrix**

3.6 Disconnected Graph

Once again comparing to the above said situation consider the remaining other two persons communicating among themselves, in another separate chat of two of them which disconnect them with their group. This kind of chat graph can be described as a disconnected graph, as shown in Figure 6. The adjacency matrix is presented as in fig 6



	V_1	V_2	V_3	V_4	V_5
V_1	0	1	1	0	0
V_2	1	0	1	0	0
V_3	1	1	0	0	0
V_4	0	0	0	0	1
V_5	0	0	0	1	0

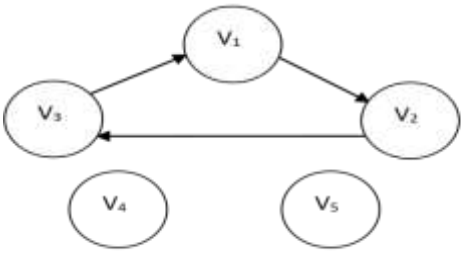
Adjacency matrix

Figure 6: A model of disconnected graph as applied to Facebook messenger group with three active members while two are idle in the group chat, yet connected on a separate chat

3.7 Influence Graph

In studies of group behaviour, it is observed that certain people can influence the thinking of others. A directed graph called an **influence graph** can be used to model this behaviour. Once again referring to a Facebook messenger group of five persons in which three persons are communicating between themselves and the messages are directed to a particular person amongst them. When a group chat is inactive or idle and the group members are participating in a separate chat with each other as in fig.7. Suppose from the graph below that V_1 sends a picture message to V_2. V_2 opens the message and watches it and in turn, sends it to V_3, V_3 open the message and send the same to V_1 as shown in fig 7, this kind of connection is a typical directed graph model, this can also be called an influence graph. Again taking note of the situation stated above, it can be referred to as an influence graph as shown in Figure7.

The adjacency matrix is presented as in fig.7



Adjacency matrices

	V1	V2	V3	V4	V5
V1	0	1	0	0	0
V2	0	0	1	0	0
V3	1	0	0	0	0
V4	0	0	0	0	0
V5	0	0	0	0	0

Figure 7 A model of an influence graph as related to a Facebook messenger group where all five persons are inactive in the group, but three are active in a separate chat with each other.

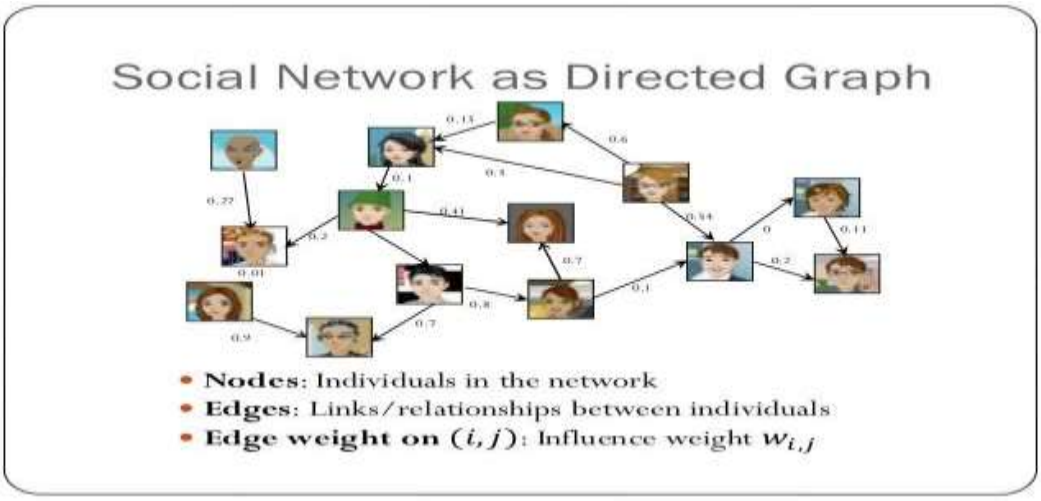


Fig. 8 Model of a Social Network as directed graph

As in fig. 8 the model terms individuals in the network as Nodes and the relationships between this individuals or link is called edges and edge weight between any two individuals is their influence weight.

3.8 Facebook Friendship as related to Graph theory

A central function of Facebook, and every other social network, is to connect people. Facebook disseminate messages for billions of users every day, [5]. The real world connections that exist between friends and acquaintances are recorded on the Web, helping us to maintain those relationships, discover friends of friends, and define our personal network. Social networks are an excellent representation of the connections that can exist between people, but the great insight from the Facebook friend graph is that the same methods that help us describe and understand a social network are also effective in understanding other kinds of data.

Imagine every person on Facebook is a node, and every like, tag comment, or share is an edge.

Facebook can do lots of statistics on their data with graph theory, it's really fun since graphs can be represented as matrices, linear algebra and statistics enables one to study distributions of connections (i.e. "friends") percolation (e.g. "unfriending"), clustering (e.g. people with

most number of friends), associativity (i.e. our tendency to connect with people who are similar to us) and many more. They can create random graphs, simulate processes happen within the network and predict outcomes. When we extend the graph to a much larger group the result is much more impactful. The individual connections begin to make up not just friendships, but friend groups and entire clusters representing the larger social network. These graphs are a great way to visualize how people relate to each other, and how those relationships sit within the greater picture of the entire social network. The graph is not only a visual representation of relationships, but it's also a mathematical one, and that allows us to read even more into the social graph. The connections between two, three, four or more people don't just tell us about that cluster, but their connections inform the potential connections between people we have yet to connect.

3.9 Graph Model of Facebook Post Reactions

In this section we shall take a look at how a model of Facebook posts and users' reactions to them. As some of you might know, there are six different types of reactions available on Facebook today: (i) Like (ii) Love (iii) Haha (iv) Wow (v) Sad (vi) Angry
There are a couple of different ways how this can be modelled. We shall here use three graph models to describe peoples' response to a post on a Facebook.

3.9.1 Reaction Type as an Attribute of a Relationship

Graph Model I

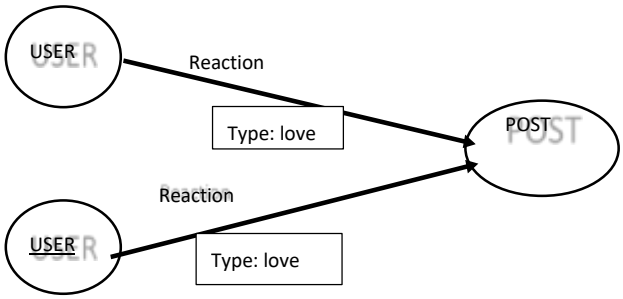


Fig. 9 *Graph model of posts and reaction types*

Graph model of posts and reaction types in all graph the three models. There will be nodes labelled users and post. In the First graph model we store the reaction type as an attribute of relationship between a user and a post

Graph Model II

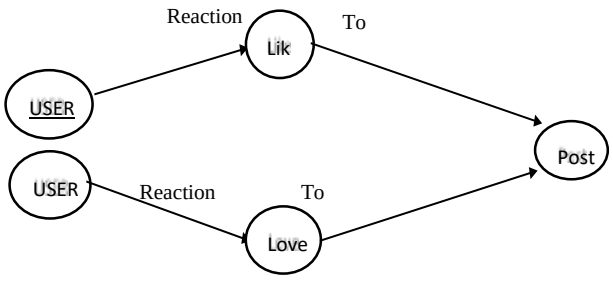


Fig. 10 *Graph Model of Reaction Type as a Node Label*

Here, we introduce an intermediary node between the user and the post. The reaction type will be stored as a label of the intermediary node. This has no other purpose rather it will always have one incoming and one outgoing relationships.

Graph Model III

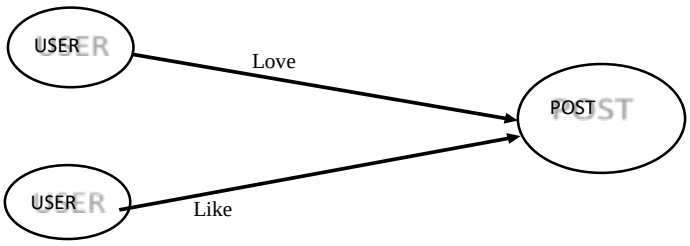


Fig: 11 *Reaction Types as a Relationship Types*

As the title mention, we will store the reaction type as a relationship between user and a post

4.0 Mathematical Expression of Relations to Facebook Friendship

Facebook friendship is a relationship between elements of sets. Relationships between elements of sets are represented using the structure of a relation [6]. A relationship between elements of two sets is expressed using ordered pairs or binary relations. Binary relation from *A* to *B* is a set of **R** of ordered pairs where the first element of each ordered pair comes from set *A* and the second element comes from set *B*. We use *A***R***B* to denote that *A* is a Facebook friend of *B*. To describe Facebook friendship, we need to fulfil certain relational properties [6]:

1. An element is always related to itself or a relation **R** on a set *A* is called reflexive if *a***R***a*for every element $a \in A$.
2. If an element *a* is related to an element *b*, and *b* is in turn related to an element *c*, then *a* is also related to *c*; that is, a relation **R** on a set *A* is called transitive if whenever *a***R***b*and *b***R***c*, then *a***R***c*, for all *a; b; c* $\in A$.If *a***R***b*, then *b* is related by a symmetric relation **R** to *a* or on a set *A*if *b***R***a*whenever *a***R***b*for all (*a; b*) $\in A$.
3. If every element in *a*also is in *b* and every element in *b* is in *a*, then *a* and *b* must be equal or an antisymmetric relation **R** on a set *A* such that for all (*a; b*) $\in A$, if *a***R***b*and *b***R***a*, then $a = b$.

In our context, there is no reflexive relation in Facebook friendship. Conversely, transitivity holds in certain situations but not always. Facebook friendship seldom falls to the category of antisymmetric relation due to the fact that it is rare for two friends to have identical friends. For a transitive relation, *A***R***B* and *B***R***C*, then *A***R***C*. It is likely for *A* to be friends with *C* if they share a common friend *B*. But it is not always possible for *A* and *C* to be friends. In reality, friendship is often intransitive. Even though friendship is not necessarily transitive, it can be viewed as a symmetric relation. In a symmetric relation, if *A* is related by **R** to *B*, then *B* is related by **R** to *A*, which can be true for Facebook friendship. Therefore, we can model the Facebook friendship by using a set of users and a binary relation of friendship which is irreflexive and symmetric. Friendship is a symmetric relation but the way in which the attitudes are shaped by the community is asymmetric [4]. The reason is simple different people have different access to information and processes. Similarly, the concept of a social group is to build up a community that is based upon an interest, same view, likeness or dislikeness or some kind of association. In this paper, we consider a "Facebook group" as a set of friends closed under the asymmetric relation of friendship.

4.1 Models based on messages Post on Facebook

When messages are post through Facebook, one can control who received the message and who should not receive the message. The models below are presented as to represent messages post to different group of friends as appear in section 1.4, features of Facebook.

I. Only Me

Fig. 12 Model of message post only to the sender (A Null graph)

This post can only be seen by the Facebook user who posts the message himself. This is represented by a node and can simply be call a null graph. This kind of graph is known as reflexive relation as in 4.0 (1) above.

II, **Acquaintance Friends only**

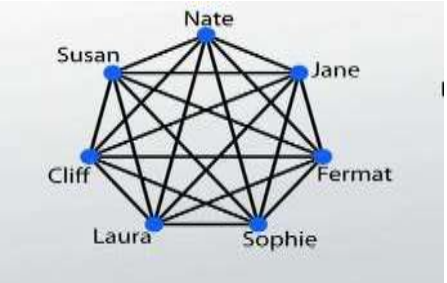


Fig. 13 A Model of message post to Facebook acquaintance friends only

This kind of message is received and seen by Nate, Susan, Cliff, Laura, Sophie, Fermat and Jane only those who are tag friends of the one who post the message. The above model suppose Sophie post a message on her walls on Facebook and tag the message to friends only, suppose the model is that of only acquaintance friends to Sophie as above, this message will only be seen by these set of friends and non-other. This kind of relationship is transitive relation

III **Friends excerpt**

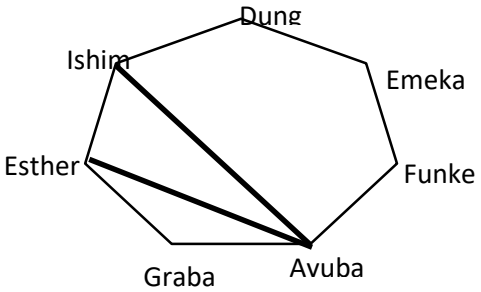


Fig. 14 A Model of message post to Facebook friends Excerpt

When messages are posted on Facebook walls, the Facebook user who posts this message can determine who can and cannot see the message sent. The model explain, suppose that Ayuba post a message on his Facebook wall and while posting he command that this message is to all friends excerpt Dung and Emeka , so only Garba, Esther, Ishim and Funke can read the message. The model appears as in fig. 13 above.

IV **Public**

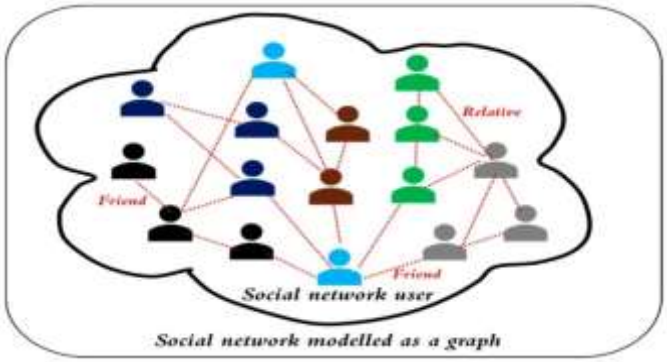


Fig. 15 **Model of message post to the public**

This model demonstrated that Posting messages on Facebook public walls- allow all friends to assess the message, this include acquaintance, relative, close friends, friends etc.

V. **Friends**

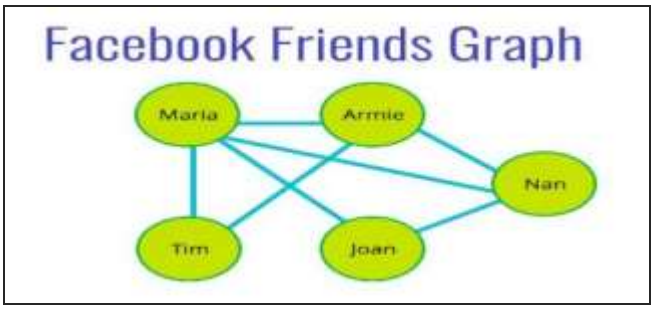


Fig 16 A Model of Facebook friends as undirected graph

An undirected graph is when each node has a reciprocal connection. So, you could say A is connected to B and B is connected to A. A real world example of this is when you add a friend on Facebook. Each user now has full access to the other user's public content.

SUMMARY AND CONCLUSION

4.3 Summary

The field of mathematics plays a vital role in the various types of fields. Many millions of people on an adjustable basis use online social networking (OSN) sites such as Facebook, Twitter, WhatsApp, and Myspace. Social media is very prevalent among youth-adult over the last decade. It has a high influence on students. The social media and custom usage of social media has an unintended effect on social communication and interpersonal relationships and self-concept among college students.

Social media networks also have become a part of our life, in current years. In this preliminary work paper, our purpose is to analyse Facebook, considering relationships among subscribed users and relating it with some models of the graph. It also describes the model of formation and properties of the social network via a graph theoretical approach and also considers the interaction between different sets of people in a social network. It also describes additional information about each individual in a network. We have also described the matrix representation of the social network. The purpose of this treatise is to apply graph theory with social media networking. It also explains how two persons or more are related to

each other and how they can access each other's profile. A social network is a social structure made up of individuals (or organizations) called "nodes or vertices", which are tied (connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, dislike, sexual relationships, or relationships of beliefs, knowledge or prestige.

3.4 Conclusion

In this work, we focused on extracting the relevant information about relationships on social media networks precisely Facebook and Facebook messenger groups. Thus the relationship in Facebook and Facebook messenger has been discussed and also, modelled with the concept of graph theory. These techniques work on any graph, including the social graph. When you have a graph as complete you're able to do a lot of interesting stuff. Imagine a marketer who wants to have a sponsored news feed item Facebook can charge a premium because they can target the influencers by using techniques like the ones above. Of course, I can't say whether Facebook is using some, none, or all of the techniques I described. But that doesn't mean application developers can't. By keeping track of who influences who you can use these techniques to maximize your exposure.

Thus the Facebook models of the graph have been discussed and also, related to the concepts of graph theory. The methodological focus of this research concentrated on relevant areas of operations research, including graph theory, network models that were shown to add insight into the analysis of social networks. The techniques developed in this paper, extend the existing operations research methods to the social network as model graphs.

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